ON THE TRANSPORT OF QUASI-GEOSTROPHIC POTENTIAL VORTICITY [1]

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ABSTRACT

The meridional transport of quasi-geostrophic potential vorticity is calculated from atmospheric data from 1 yr. The calculation is based on available data for the transports of relative momentum and sensible heat from eight standard pressure levels. The results show that the transport of potential vorticity is negative (southward) in the major part of the troposphere above the lowest layer of the atmosphere. A small region of positive (northward) transport is found in connection with the subtropical jet stream in summer.

The computed transports of potential vorticity are used in a calculation of the mean annual heat sources in the atmosphere from a steady state quasi-geostrophic model. The results show that the atmosphere is heated south of 50°N and cooled north of this latitude and that the major heat source and heat sink are found around 70 cb. The calculation compares favorably with the determination of the heat sources from heat budget calculation.

It is shown that the transport of potential vorticity is along the gradient of the potential vorticity in the troposphere above approximately 80 cb. Exchange coefficients for the transport of potential vorticity are computed as a function of latitude and pressure for the annual mean values. In addition, exchange coefficients for the transport of sensible heat are obtained, and it is shown that these coefficients are positive in the troposphere below 20 cb.

1. INTRODUCTION

During the last two decades, there has been a considerable interest in the meridional transport of various quantities in the atmosphere in connection with a description of the atmospheric general circulation. The meridional transport of sensible heat and relative momentum are probably the two quantities that have been investigated in the greatest detail, but other quantities such as the transport of moisture have also received considerable attention. An excellent summary of numerous studies of transport processes has been given by Lorenz (1967).

In connection with certain theoretical studies of the general circulation of the atmosphere, it appears desirable to parameterize some transport processes. This type of approach has especially been used by Saltzman (1968), Green (1969), and Wiin-Nielsen (1970). While it is possible to use a relatively simple approach in connection with the transport of sensible heat to describe the main characteristics of this process, it is much less straightforward to parameterize the momentum transport because the observed

momentum transport in general is from regions of low relative momentum to regions of high relative momentum. Saltzman and Vernekar (1968) have given one procedure that can be used to parameterize the momentum transport when the seasonal variation has been removed. This approach, therefore, cannot be applied in studies where we are interested in a description of the annual variation of the general circulation. Another idea dealing with the parameterization of the momentum transport has been expressed by Green (1969) who assumes that a diffusion approximation can be made for the meridional transport of the quasi-conservative quantities of entropy and potential vorticity. It is the purpose of this paper to investigate some aspects of this approach to the momentum transport.

To the knowledge of the authors, the transport of potential vorticity has not been calculated from atmospheric data. In view of the fact that some of the results of the present study will be used in a theoretical study of the annual variation of the general circulation based on a quasi-geostrophic model, it appears natural to restrict our study to the transport of the quasi-geostrophic potential vorticity from the known transports of momentum and heat. The annual mean values of the transport of

potential vorticity will be used to make a diagnostic calculation of the zonal mean of the heat sources in the atmosphere and the mean meridional circulation. The values will also be used to investigate whether or not the transport of potential vorticity can be described in terms of a diffusion process.

2. TRANSPORT OF POTENTIAL VORTICITY

The quasi-geostrophic potential vorticity Q may be written in the form

$$Q = f + \xi + \frac{\partial}{\partial p}\left[\frac{f_0^2}{\bar{\sigma}}\frac{\partial \psi}{\partial p}\right] \tag{1}$$

where $f = 2\Omega \sin \phi$ is the Coriolis parameter; ξ, the relative vorticity; $\bar{\sigma} = -(\alpha/\theta)(\partial\theta/\partial p)$, a measure of static stability; α, the specific volume; θ, the potential temperature; p, pressure; and ψ, the stream function. In this study, we have assumed that

$$\frac{\partial \psi}{\partial p} = \frac{1}{f_0}\frac{\partial \phi}{\partial p} = -\frac{1}{f_0}\frac{RT}{p}. \tag{2}$$

The quantity Q is conserved in a quasi-geostrophic model of the atmosphere in the absence of heating and friction. The special formulation of potential vorticity used in quasi-geostrophic models has been treated in detail by Phillips (1963).

In this section, we shall describe the procedure and the results of a calculation of the meridional transport of the quasi-geostrophic potential vorticity Q. Since we are working with a quasi-geostrophic model, we must assume that the wind used for advection is nondivergent. Denoting a zonal average by a subscript z defined as

$$\left(\quad \right)_z = \frac{1}{2\pi}\int_0^{2\pi}\left(\quad \right)d\lambda, \tag{3}$$

we may write that

$$(Qv)_z = (\xi v)_z + \frac{\partial}{\partial p}\left[\frac{f_0^2}{\bar{\sigma}}\left(\frac{\partial \psi}{\partial p}v\right)_z\right]. \tag{4}$$

In obtaining eq (4), we have made use of the fact that $v_z = 0$ because the wind is nondivergent and that

$$\left[v\frac{\partial}{\partial p}\left[\frac{f_0^2}{\bar{\sigma}}\frac{\partial \psi}{\partial p}\right]\right]_z = \frac{\partial}{\partial p}\left[\frac{f_0^2}{\bar{\sigma}}\left(\frac{\partial \psi}{\partial p}v\right)_z\right] - \frac{f_0^2}{\bar{\sigma}}\left(\frac{\partial \psi}{\partial p}\frac{\partial v}{\partial p}\right)_z. \tag{5}$$

It is seen that the last term in eq (5) is zero, because

$$\left[\frac{\partial \psi}{\partial p}\frac{\partial v}{\partial p}\right]_z = \left[\frac{\partial \psi}{\partial p}\frac{1}{a\cos\phi}\frac{\partial}{\partial \lambda}\left(\frac{\partial \psi}{\partial p}\right)\right]_z$$

$$= \frac{1}{2a\cos\phi}\left[\frac{\partial}{\partial \lambda}\left(\frac{\partial \psi}{\partial p}\right)^2\right]_z = 0. \tag{6}$$

Equations (5) and (6) show the correctness of eq (4).

It is obvious that the last term in eq (4) is connected with the heat transport. We may, for example, write

$$\frac{f_0^2}{\bar{\sigma}}\left(\frac{\partial \psi}{\partial p}v\right)_z = -\frac{f_0 R}{\bar{\sigma}p}(Tv)_z \tag{7}$$

using eq (2) to obtain the last relation.

The first term in eq (4) is related to the momentum transport. It can be shown that

$$(\xi v)_z = -\frac{1}{a\cos^2\phi}\frac{\partial(uv)_z\cos^2\phi}{\partial \phi}. \tag{8}$$

The proof of the relation (8) is based on the fact that the wind vector (u, v) is nondivergent and therefore satisfies the relation

$$\frac{1}{a\cos\phi}\left(\frac{\partial u}{\partial \lambda} + \frac{\partial v\cos\phi}{\partial \phi}\right) = 0. \tag{9}$$

We may prove eq (8) in the following way:

$$(\xi v)_z = \frac{1}{a\cos\phi}\left(\left(\frac{\partial v}{\partial \lambda} - \frac{\partial u\cos\phi}{\partial \phi}\right)v\right)_z$$

$$= -\frac{1}{a\cos^2\phi}\left(v\cos\phi\frac{\partial u\cos\phi}{\partial \phi}\right)_z. \tag{10}$$

From the last relation, we get

$$(\xi v)_z = -\frac{1}{a\cos^2\phi}\frac{\partial(uv)_z\cos^2\phi}{\partial \phi} + \frac{1}{a\cos\phi}\left(u\frac{\partial v\cos\phi}{\partial \phi}\right)_z$$

$$= -\frac{1}{a\cos^2\phi}\frac{\partial(uv)_z\cos^2\phi}{\partial \phi} - \frac{1}{2a\cos\phi}\left(\frac{\partial u^2}{\partial \lambda}\right)_z$$

$$= -\frac{1}{a\cos^2\phi}\frac{\partial(uv)_z\cos^2\phi}{\partial \phi}. \tag{11}$$

Using eq (7) and (8), we may write eq (4) in the form

$$(Qv)_z = -\frac{1}{a\cos^2\phi}\frac{\partial(uv)_z\cos^2\phi}{\partial \phi} - \frac{\partial}{\partial p}\left[\frac{f_0 R}{\bar{\sigma}p}(Tv)_z\right]. \tag{12}$$

It is seen from eq (12) that $(Qv)_z$, the meridional transport of quasi-geostrophic potential vorticity, may be evaluated in a straightforward manner if we have data for the momentum transport $(uv)_z$ and the heat transport $(Tv)_z$. Such data are available from several sources, but it is only in a few cases that we have data on the two quantities computed on exactly the same basic data. We shall later consider the transport $(Qv)_z$ based on momentum and heat transport data from the year 1963. However, before we consider the details of the calculations, it may be useful to consider what we can expect regarding the sign of $(Qv)_z$. We note first that the factor $\bar{\sigma}p$ is rather constant through the troposphere. This is due to the fact that the vertical variation of $\bar{\sigma}$ empirically is reasonably well described by the formula

$$\bar{\sigma} = \bar{\sigma}_0\frac{p_0}{p}. \tag{13a}$$

The sign of the last term in eq (12) depends therefore mainly on the vertical variation of the heat transport. It is known from several investigations [e.g., Wiin-Nielsen et al. (1963, 1964)] that $(Tv)_z$ at most latitudes has a maximum in the lower part of the troposphere; and we would therefore expect that the last term in eq (12) will give a negative contribution at most latitudes and at most pressure levels above the maximum in the heat transport. On the other hand, the first term in eq (12) will
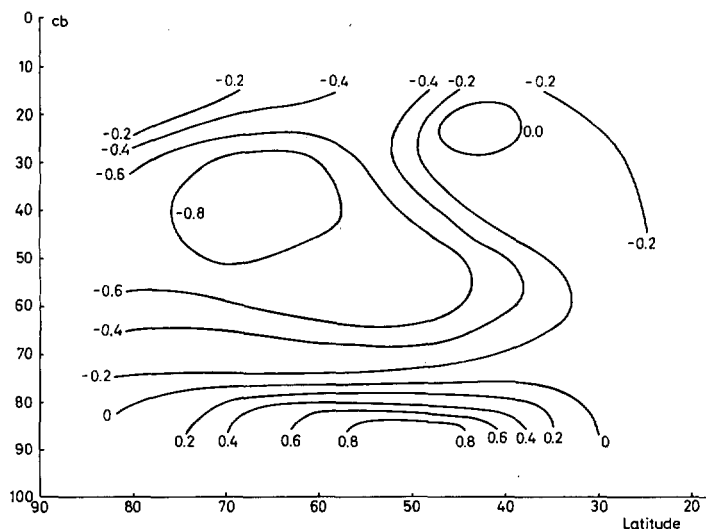
FIGURE 1.—Annual mean of the meridional transport of quasi-geostrophic potential vorticity as a function of latitude and pressure; units, 10^{-4} m/s².
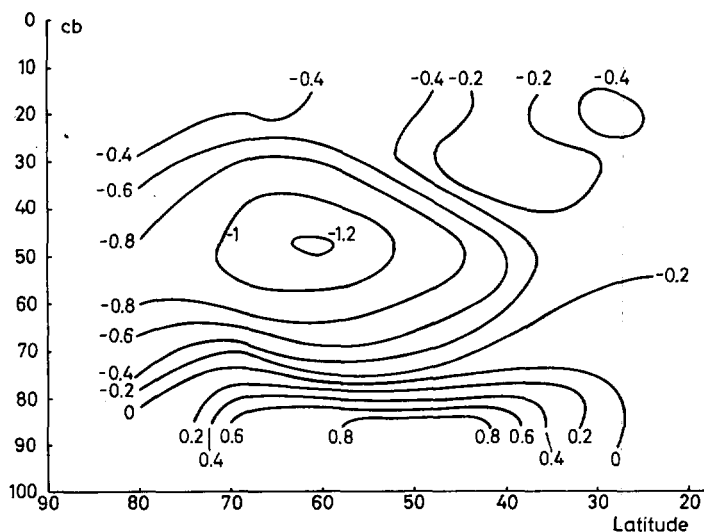

FIGURE 2.—Same as figure 1, but for the winter. The data were obtained as average for the year 1963 months of January, February, March, October, November, and December.

at most pressure levels give a positive contribution in the middle latitudes, where we observe a convergence of the momentum transport, while the contribution will be negative in the low and the high latitudes. We would expect (based on this knowledge) a negative (southward) transport $(Qv)_z$ in the low and high latitudes, while the sign of $(Qv)_z$ in the middle latitudes is questionable because the two terms in eq (12) oppose each other. To settle this question and furthermore to know the magnitude of $(Qv)_z$, we must make detailed calculations.

The data used for the calculations consist of momentum and heat transport data for each of the months of the year 1963. The data for the heat transport were available for each of the layers between the standard pressure levels 100, 85, 70, 50, 30, 20, 15, and 10 cb. If we ascribe these heat transports to the middle of the layers, we can compute the second term in eq (12) at the levels from 85 to 15 cb. Since the momentum transport is available at the same levels, we can compute $(Qv)_z$ at these levels. Since the original transport data were available from 22.5°N to 82.5°N, we will get $(Qv)_z$ from 25°N to 80°N. The main features of the transport of the quasi-geostrophic potential vorticity can be seen from figure 1 that shows the transport averaged for the whole year 1963 in the unit 10^{-4} m/s².

It is seen that $(Qv)_z$ is negative from 70 to 15 cb, with the exception of a small region around 40°–45°N and at 20 cb. The maximum negative transport is found between 50 and 30 cb in the region around 70°N. The transport $(Qv)_z$ is positive almost everywhere at 85 cb. It is easily seen from the original data that this is due to the fact that the heat transport is larger in the layer 85 to 70 cb than in the layer from 100 to 85 cb. Physically, we may ascribe this to the conditions in the atmospheric planetary boundary layer.

Figures 2 and 3 show the transport $(Qv)_z$ for the winter and the summer season, respectively. The winter season
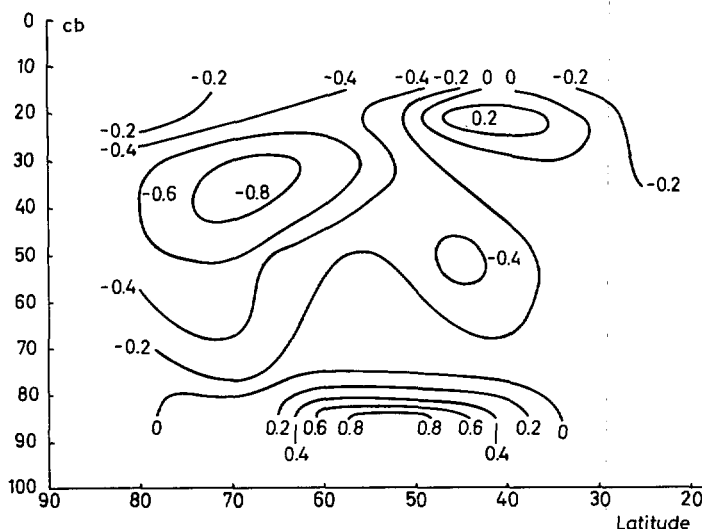

FIGURE 3.—Same as figure 1, but for the summer. The data were obtained as average for the months of April, May, June, July, August, and September.

is defined as the average of the months January, February, March, October, November, and December, while the summer season in this study is the average for the remaining months. The main features of the two distributions are the same. The transport is predominantly negative above approximately 80 cb, while it is positive below this level. The maximum southward transport is somewhat larger in the winter season and located at a lower level than in the summer. It is also seen that the northward (positive) transport which is found in the annual mean (at 20 cb and 40° – 45°N) is caused by the positive transport in the summer.

It is, in general, possible to gain some further insight into transport processes in the atmosphere by considering the balance requirements. Such a consideration does not

give an explanation of why the transport exists, but it only says what transport is necessary to satisfy a long-term balance. Lorenz (1967) has stressed this point repeatedly in his discussion of the balance requirements for water, heat, and momentum. By considering the balance requirements for potential vorticity, we are thus unable to answer the question of why the transport of potential vorticity is as calculated; but we may relate the transport of potential vorticity to the balance requirements for heat and momentum. This will be attempted in the following paragraphs.

In trying to establish the relation between the transport of potential vorticity and other quantities, we shall make use of the well-established fact that there is a net heating in low latitudes and net cooling in high latitudes. When we combine this statement with our knowledge of the temperature distribution in the atmosphere, we may also express the statement as follows: There is a positive generation of zonal available potential energy in the atmosphere, that is,

$$G(A_z) > 0. \tag{13b}$$

Restricting our considerations to the time-averaged state of the atmosphere, we find using the quasi-geostrophic equations that the steady-state equation for potential vorticity is

$$\nabla \cdot (Q\mathbf{V}) = -f_0 \frac{R}{c_p} \frac{\partial}{\partial p} \left(\frac{H'}{\bar{\sigma}p} \right) \tag{14}$$

where Q and $\bar{\sigma}$ are defined in eq (1) and H' is the diabatic heating per unit mass and unit time.[2] Equation (14) applies for the free atmosphere and is based on the assumption that the major frictional dissipation is confined to the atmospheric boundary layer.

By taking the zonal average of eq (14), we obtain

$$\frac{1}{a \cos \phi} \frac{\partial (Qv)_z \cos \phi}{\partial \phi} = -f_0 \frac{R}{c_p} \frac{\partial}{\partial p} \left(\frac{H'_z}{\bar{\sigma}p} \right). \tag{15}$$

We may now use eq (15) to evaluate the expression for $G(A_z)$ that is

$$G(A_z) = \frac{1}{4\pi a^2 g} \int_0^{p_0} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \int_0^{2\pi} \frac{R}{c_p} \frac{1}{\bar{\sigma}p} \alpha'_z H'_z a^2 \cos \phi \, d\lambda d\phi dp \tag{16}$$

where $G(A_z)$ has been written for the whole globe and where α'_z is the deviation of the zonal average of specific volume from its area average. Using the hydrostatic equation and evaluating the integral with respect to longitude, we get

$$G(A_z) = -\frac{1}{2g} \int_0^{p_0} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{\partial \Phi'_z}{\partial p} \left(\frac{RH'_z}{c_p \bar{\sigma}p} \right) \cos \phi \, d\phi dp. \tag{17}$$

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[2] A prime denotes the deviation from the area mean of the quantity.

We may integrate eq (17) by parts with respect to pressure and obtain

$$G(A_z) = -\frac{1}{2g} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{R}{c_p} \frac{1}{\bar{\sigma}_0 p_0} \Phi'_{z0} H'_{z0} \cos \phi \, d\phi$$
$$+ \frac{1}{2g} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \int_0^{p_0} \Phi'_z \frac{R}{c_p} \frac{\partial}{\partial p} \left(\frac{H'_z}{\bar{\sigma}p} \right) \cos \phi \, dp d\phi. \tag{18}$$

Inserting from eq (15) in eq (18), we get

$$G(A_z) = -\frac{1}{2g} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{R}{c_p} \frac{1}{\bar{\sigma}_0 p_0} \Phi'_{z0} H'_{z0} \cos \phi \, d\phi$$
$$- \frac{1}{2g} \int_0^{p_0} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{\Phi'_z}{f_0 a} \frac{\partial (Qv)_z \cos \phi}{\partial \phi} d\phi dp. \tag{19}$$

The last integral in eq (19) may again be integrated by parts with respect to latitude; and by using the geostrophic wind relation, we obtain

$$G(A_z) = -\frac{1}{2g} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{R}{c_p} \frac{1}{\bar{\sigma}_0 p_0} \Phi'_{z0} H'_{z0} \cos \phi \, d\phi$$
$$- \frac{1}{2g} \int_0^{p_0} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} (Qv)_z u_z \cos \phi \, d\phi dp. \tag{20}$$

The first integral in eq (20) is negative as we shall demonstrate a little later. To satisfy the inequality (13b), one must be certain that the last integral in eq (20) be positive, or, in other words, that there exists a negative correlation between $(Qv)_z$ and u_z. In view of the fact that u_z is positive in the major part of the meridional cross section, it follows that $(Qv)_z$ must be essentially negative to satisfy eq (13b). In short, it is a necessity that $(Qv)_z$ is mainly negative in the free atmosphere to be in agreement with the fact that we observe a net heating in low latitudes and a net cooling in high latitudes.

The argument above rests on the assumption that the first integral in eq (20) is negative or that

$$\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{R}{c_p} \frac{1}{\bar{\sigma}_0 p_0} \Phi'_{z0} H'_{z0} \cos \phi \, d\phi > 0. \tag{21}$$

We may show that eq (21) is satisfied by considering the steady-state thermodynamic equation at the lower boundary. We obtain

$$\frac{1}{c_p} H'_{z0} = \frac{1}{a \cos \phi} \frac{\partial (Tv)_{z0} \cos \phi}{\partial \phi} - \frac{\bar{\sigma}_0 p_0}{R} \omega_{z0}. \tag{22}$$

Inserting from eq (22) in the integral in eq (21), we get

$$\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{R}{c_p} \frac{1}{\bar{\sigma}_0 p_0} \Phi'_{z0} H'_{z0} \cos \phi d \phi$$
$$= \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{R}{a \bar{\sigma}_0 p_0} \Phi'_{z0} \frac{\partial (Tv)_{z0} \cos \phi}{\partial \phi} d\phi - \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \Phi'_{z0} \omega_{z0} \cos \phi \, d\phi. \tag{23}$$

We shall now assume that the vertical velocity at the lower boundary is caused mainly by the frictional stresses or, in other words, that

$$\omega_{z0} = -\frac{g}{f} \frac{1}{a \cos \phi} \frac{\partial \tau_\lambda \cos \phi}{\partial \phi}. \qquad (24)$$

Assuming that

$$\tau_\lambda = -C_D \rho_0 V_0 u_0, \qquad (25)$$

we get

$$\omega_{z0} = \frac{g}{f} \frac{C_D \rho_0 V_0}{a \cos \phi} \frac{\partial u_{z0} \cos \phi}{\partial \phi}. \qquad (26)$$

The expression (26) is substituted in eq (23), both integrals are integrated by parts; and we obtain

$$\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{R}{c_p} \frac{1}{\sigma_0 p_0} \Phi'_{z0} H'_{z0} \cos \phi \, d\phi = \frac{R f_0}{\sigma_0 p_0} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} (Tv)_{z0} u_{z0} \cos \phi \, d\phi$$

$$- g C_D \rho_0 V_0 \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} u_{z0}^2 \cos \phi \, d\phi. \qquad (27)$$

The last integral in eq (27) is obviously negative. The first integral is positive, because $(Tv)_{z0}$ is mainly positive, but small where u_{z0} is negative in the low and high latitudes and large where u_{z0} is positive in the middle latitudes. It will be shown in the next section that the first term in eq (27) is numerically larger than the second. Equation (21) is therefore satisfied.

A simpler, but less complete, analysis of the balance requirements can naturally also be obtained from eq (15) by integrating this equation from the top of the atmosphere to an arbitrary pressure level p. Denoting

$$I(p) = \int_0^p (Qv)_z dp, \qquad (28)$$

we get

$$f_0 \frac{R}{c_p} \frac{1}{\bar{\sigma} p} H'_z(p) = -\frac{1}{a \cos \phi} \frac{\partial I(p) \cos \phi}{\partial \phi}. \qquad (29)$$

In obtaining eq (29), we have used the boundary condition that $(\bar{\sigma} p)^{-1} H'_z \to 0$ for $p \to 0$. This is justified because $\bar{\sigma}$ goes to infinity more rapidly than p^{-1}.

In view of the fact that H'_z is positive in the low latitudes and negative in the high latitudes, it follows that we must have convergence of the integrated transport in the low latitudes and divergence of the integrated transport in the high latitudes. A comparison with the distributions presented in figures 1, 2, and 3 shows that this requirement also is satisfied.

3. CALCULATION
OF THE ZONALLY AVERAGED HEATING

The results of the calculations of the transport of potential vorticity described in section 2 can be used to calculate the zonal average of the atmospheric heating. It was shown in the preceding section that the transport in a qualitative sense satisfies the requirement created by the heating of the atmosphere. Unfortunately, we do not have detailed information on the distribution of the diabatic heating due to all physical processes as a function of latitude and pressure. Calculations of this type have been carried out by Wiin-Nielsen and Brown (1962) and Brown (1964); but these calculations were based on information from only two vertical pressure levels, and only one value of heating was obtained in each vertical column. In the studies by Lawniczak (1969), the calculations were extended to cover all standard-pressure levels between 100 and 10 cb, but these calculations covered only 1 mo. The present calculations of the potential vorticity transports can be used to obtain the zonally averaged heating field in the average over a period where we may assume that the conditions for a steady state apply.

The calculation of H_z is based on eq (15). Since the transport $(Qv)_z$ is known only to the 15-cb level, it is advisable to integrate eq (15) from the surface to an arbitrary pressure level. Denoting

$$J(p) = \int_p^{p_0} (Qv)_z dp, \qquad (30)$$

we get

$$H_z = \frac{\bar{\sigma} p}{\sigma_0 p_0} H_{z0} + \frac{c_p}{R} \frac{1}{f_0} \frac{\bar{\sigma} p}{a \cos \phi} \frac{\partial}{\partial \phi} (J(p) \cos \phi) \qquad (31)$$

from eq (15).

The second term in eq (31) can be calculated directly from the data on the transport $(Qv)_z$. To calculate H_{z0}, we make use of eq (22) and (26), which require a knowledge of the heat transport at the surface and the zonally averaged winds at the same level.

The data which have been used to calculate the various terms are the same as those applied for the transport calculations described in section 2. We shall first describe the results of the calculation of the first term in eq (22). The data were the values of $(Tv)_z$ in the layer 100 to 85 cb. The values of the term given in the unit of deg./day is shown in figure 4 as a function of latitude. By using the annual average of u_{z0} obtained from the 100 cb data, the second term in eq (22) was evaluated. The result of this calculation is shown in figure 5. As one can see directly from eq (26), we can expect to find sinking motion $(\omega_{z0} > 0)$ south of the maximum in u_{z0} and rising motion $(\omega_{z0} < 0)$ north of the maximum westerlies. Figure 5 shows also that the numerical values of the second term in eq (22) in general is much smaller than the first term shown in figure 4. The heating H_{z0}, which is the difference between the curves in figures 4 and 5, is shown in figure 6. We obtain heating south of 50°N and cooling north of this latitude. The maximum cooling is about 0.75 deg./day, while the maximum heating is 0.30 deg./day. It should be mentioned that, in the evaluation of ω_{z0}, we have used the standard values $C_D = 3 \times 10^{-3}$, $\rho_0 = \text{kg m}^{-3}$, $V_0 = 10$ m/s.

The calculation of $J(p)$ was carried out for the various pressure levels by replacing the integral in eq (30) by a finite sum. The values of $J(p)$ were then used to evaluate
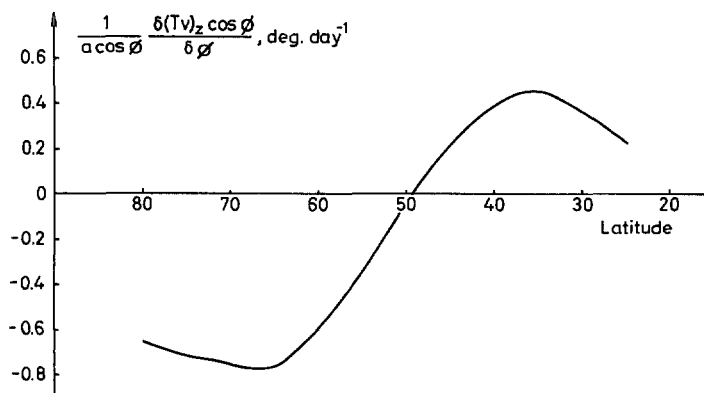
FIGURE 4.—Divergence of the heat transport at 100 cb in the average for the year 1963 as a function of latitude; units, deg./ day.
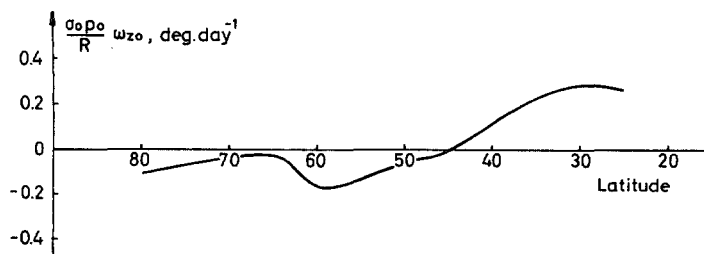

FIGURE 6.—Zonal heating at 100 cb in the average for the year 1963 as a function of latitude; units, deg./day.

FIGURE 5.—Quantity $R^{-1}\sigma_0 p_0 \omega_{z0}$ at 100 cb in the average for the year 1963 as a function of latitude; units, deg./day.

FIGURE 7.—Zonal heating in the average for the year 1963 as a function of latitude and pressure; units, deg./day.

the last term in eq (31) by finite differences. The standard values of $\bar{\sigma}=\bar{\sigma}(p)$ used in the evaluation of eq (31) were those used by Lawniczak (1969), based on the statistics of Gates (1961). The heating $H_z=H_z(\phi, p)$ is shown in figure 7, again using the unit deg./day. We observe that the separation between heating and cooling is at about 50°N. The maximum heating is about 0.6 deg./day, while the largest value of the cooling is −0.8 deg./day. The largest values of heating and cooling are found in the layer between 85 and 70 cb.

The vertical mean value of the atmospheric heating is given in figure 8 as a function of latitude. They were obtained as a weighted average from the values given in figure 7. The values given in figure 8 may be compared with independent estimates of the atmospheric heat budget. The values of $\tilde{H}_z$ given by Palmén and Newton (1969) are indicated on figure 8 by circles. In comparing the two calculations, it must be remembered that the calculation presented here is based on data from a single year and that our values of $\tilde{H}_z$ are those necessary to satisfy the steady state quasi-geostrophic equations. It is seen that both calculations result in cooling north of 50°N and heating south of this latitude. While there is reasonably good agreement between the calculations for the region of cooling, it is seen that our heating values are larger than those given by Palmén and Newton (loc. cit.). The reason for this may possibly be that the mean merid-
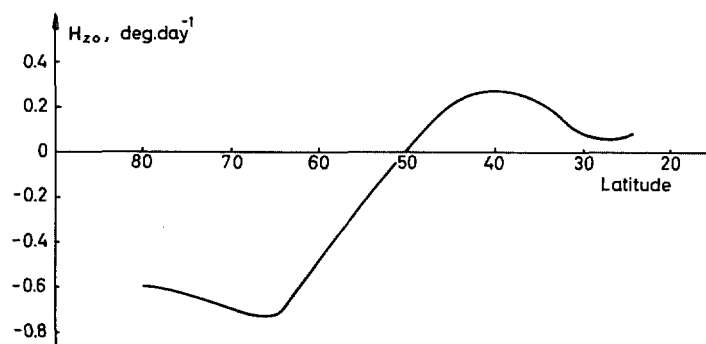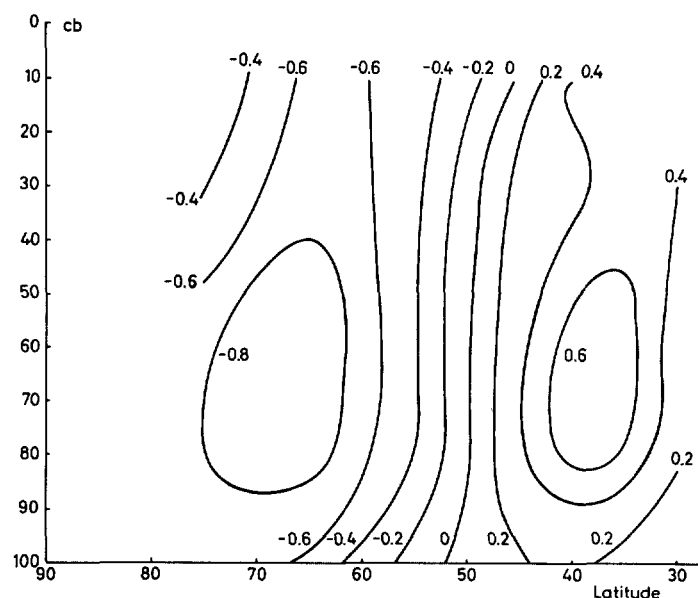
ional circulation inherent in a quasi-geostrophic model is too weak. We may see this by considering the steady-state thermodynamic equation

$$\frac{1}{c_p} H_z' = \frac{1}{a \cos \phi} \frac{\partial (Tv)_z \cos \phi}{\partial \phi} - \frac{\bar{\sigma} p}{R} \omega_z. \qquad (32)$$

If ω_z were larger in the region 30°N to 50°N where we have sinking motion ($\omega_z > 0$), it would result in a reduced value of H_z'. A similar argument applied to the region 50°N to 70°N, where we have rising motion ($\omega_z < 0$), would reduce the cooling in the region. Both calculations have a considerable amount of uncertainty, and the agreement is probably as good as can be expected. It should be pointed out that a general agreement between the two types of calculations is further indicated by the results obtained by Lawniczak (1969) who has calculated the heat sources for January 1969, using essentially the same model as the one
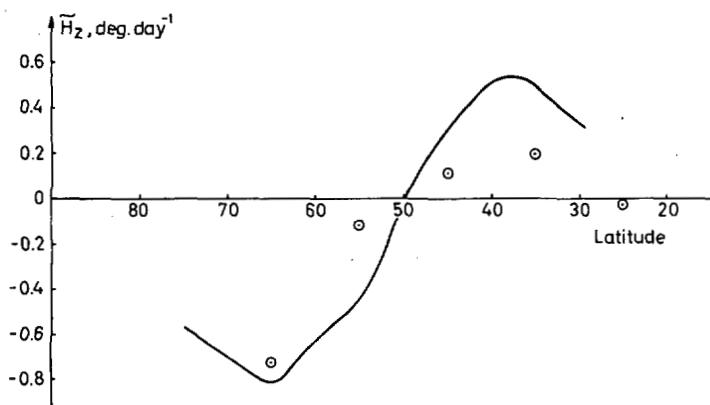
FIGURE 8.—Vertical average of the zonal heating as a function of latitude. The circles indicate the values obtained for the annual mean by Palmén and Newton (1969); units, deg./day.

FIGURE 9.—Zonal average of the vertical velocity ω_z as a function of latitude and pressure, computed from the data for 1963; units, 10^{-6} cb/s.

applied in this paper except that he naturally could not use a steady-state condition for a single month.

The mean meridional circulation can be calculated from eq (32), using the values of H'_z obtained from eq (31) and the values of the heat transport described in section 2. The result of such a calculation is shown in figure 9, where isolines for the vertical velocity in the unit 10^{-6} cb/s are drawn in the meridional plane. Since the data only permit a calculation in the latitude belt from 30°N to 75°N, we cannot obtain the low latitude Hadley circulation. However, the middle latitude Ferrel cell is clearly shown in the figure with the maximum values of ω_z at the ground at 60°N. The maximum rising motion goes from 60°N at the surface to about 40°N at 20 cb. The northern branch of the Hadley circulation in the low latitudes is clearly seen in the lower elevations in the region from 30°N to 40°N. It should furthermore be mentioned that the high latitude Hadley cell circulation is well indicated with the maximum values of ω_z found at 65°N and at 40 cb. It may be mentioned that the largest vertical velocity in the meridional cross section is about 2 mm/s. The picture of ω_z derived here has many similarities to the mean meridional circulation as shown by Starr (1966).

4. ON LATERAL MIXING APPROXIMATIONS

The concept of lateral mixing has at times played a major role in studies of the general circulation. The idea to use an exchange coefficient for the meridional transport of atmospheric properties goes back to Defant (1921) and has been used by several investigators, most recently by Adem (1964) in connection with the transport of sensible heat. In a recent study of the annual variation of atmospheric energy by Wiin-Nielsen (1970), the concept was also used to parameterize the heat transport. While a positive exchange coefficient can be used to model the transport of sensible heat, because the heat transport is from the high to the low temperatures at most elevations
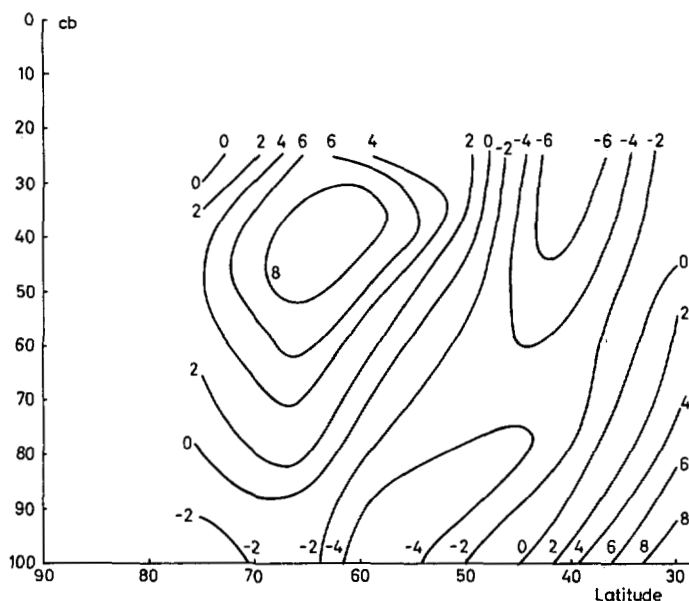
with exception of the stratosphere, it is not possible to use the same concept for the meridional transport of momentum. This transport is characteristically against the gradient of relative angular momentum. A parameterization of the momentum transport requires, therefore, more complicated procedures. In a recent paper by Green (1969), it was suggested to parameterize the transport of potential vorticity, using an exchange coefficient. If this is possible, one can obtain a parameterization of the momentum transport using eq (12) as the basic relation.

The transport of geostrophic potential vorticity has already been described in section 2 of this paper. In this section, we shall describe an investigation of how this transport is related to the meridional gradient of the geostrophic potential vorticity itself. An exchange coefficient K_Q for the geostrophic potential vorticity may be defined by the relation

$$(Qv)_z = -K_Q \frac{\partial Q_z}{a\partial\phi} \qquad (33)$$

where Q_z is the zonal mean value of Q. We know that $(Qv)_z$ is mainly negative in the atmosphere above the planetary boundary layer as shown in figures 1, 2, and 3 of this paper. If the meridional gradient of Q_z is positive, we see from eq (33) that K_Q will be a positive function. Using the definition of Q given in eq (1), we find that

$$\frac{\partial Q_z}{a\partial\phi} = \frac{2\Omega}{a}\cos\phi - \frac{1}{a^2}\frac{\partial}{\partial\phi}\left[\frac{1}{\cos\phi}\frac{\partial u_z\cos\phi}{\partial\phi}\right] - \frac{\partial}{\partial p}\left[\frac{f_0^2}{\sigma}\frac{\partial u_z}{\partial p}\right]. \qquad (34)$$

The dominating term in eq (34) is, under most atmospheric conditions, the first term which is always positive.

We may therefore expect that K_Q will be positive in the troposphere above the planetary boundary layer.

The expression (34) was evaluated from the zonal winds available for the year 1963. The calculations described in the following paragraphs were performed using the annual mean values of u_z and $(Qv)_z$. The exchange coefficient K_Q was then evaluated from the annual mean data separately in each point, using the definition (33). It is obvious from figure 1 that the concept of an exchange coefficient can not be applied at 85 cb, where $(Qv)_z$ is positive; and we shall therefore restrict the calculations to levels above 85 cb.

The values of K_Q thus derived are shown in figure 10 for the levels 15, 20, 30, 50, and 70 cb as a function of latitude, using a unit of 10^6 m²/s. Since K_Q appears everywhere in combination with cos ϕ as a factor in the dynamic equation, we have plotted K_Q cos ϕ. The value of K_Q cos ϕ has been smoothed with a smoothing operator that removes two grid-increment waves. It is seen that K_Q is positive everywhere except at 20 cb and 40°N. This shows that the transport $(Qv)_z$ in general is along the gradient of the zonally averaged potential vorticity. The general shape of the curves is the same at all levels, showing a maximum in the middle latitudes approximately at 50°–60°N and a minimum in the region 30°–40°N. The values are relatively large in the lower stratosphere (15 cb), small at 30 cb, and again larger in the middle and lower troposphere. The maximum values of K_Q cos ϕ are found at progressively lower latitudes as the pressure increases. The values of K_Q cos ϕ at 30 and 70 cb would be appropriate values to use in a two-level model, but one may naturally also form suitable mean values using the other levels.

The values of K_Q described above will be used in a model designed to simulate the annual variation of atmospheric energy. The results of this study will be reported elsewhere. In such a study, it is also necessary to have exchange coefficients for the meridional transport of sensible heat. In view of this fact, we shall describe the results of such calculations.

An exchange coefficient for the meridional transport of sensible heat may be defined in a way analogous to eq (33). We write

$$(Tv)_z = -K_H \frac{\partial T_z}{a\partial\phi}. \tag{35}$$

The available data for the year 1963 permits an evaluation of K_H for each of the layers between the standard pressure levels from 100 to 10 cb. It is well known that the transport of sensible heat is along the gradient of the zonally averaged temperature in the troposphere where K_H consequently will be positive. In the lower stratosphere, the heat transport is against the gradient of the zonally averaged temperature; and K_H will be negative. The values of K_H, in the unit 10^6 m²/s, are shown in figure 11 for the various layers that are completely in the
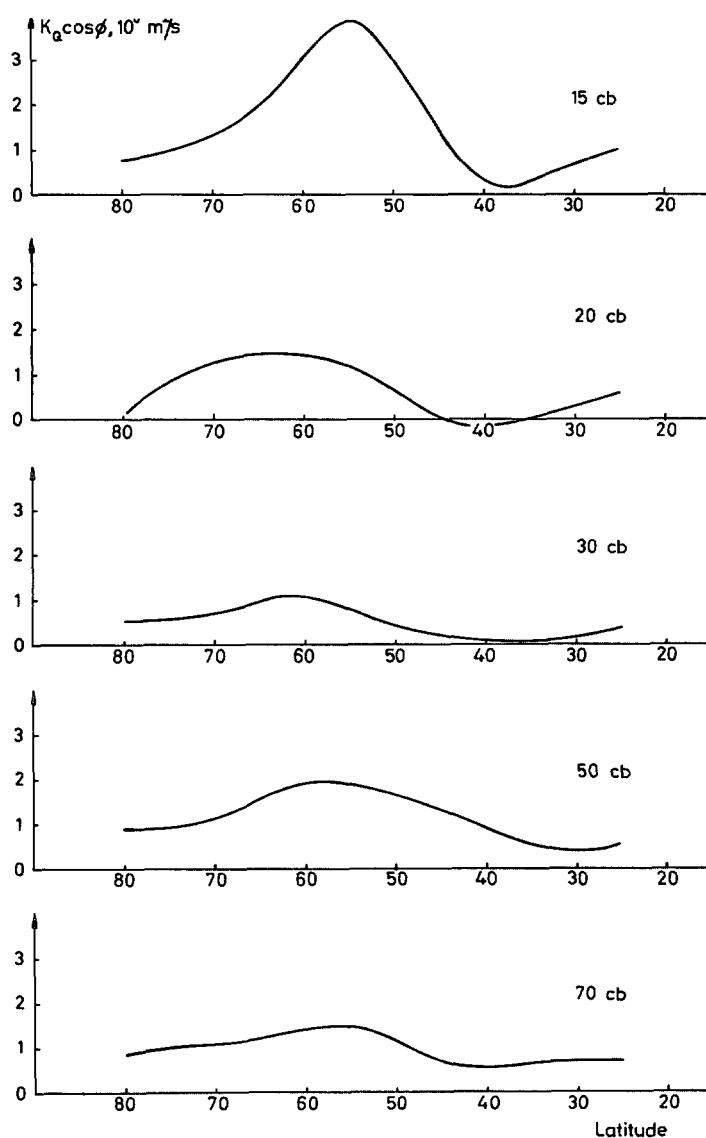

FIGURE 10.—Exchange coefficient for the transport of quasi-geostrophic potential vorticity as a function of latitude for the levels 15, 20, 30, 50, and 70 cb, computed from mean annual data; units, 10^6 m/s².

troposphere. As with K_Q, we have plotted K_H cos ϕ against latitude. The distributions show in all layers a single maximum in the middle latitudes. The numerically largest values are found in the layer from 70 to 85 cb corresponding to the maximum heat transport in this layer.

K_H was also evaluated for the layer above 30 cb, but the results are not reproduced because the model study will be restricted to the troposphere and because K_H has negative values in these layers. In the layer from 20 to 30 cb, which is a transition layer, we find positive values of K_H in the low latitudes corresponding to tropospheric conditions, while K_H is negative in the high latitudes, approximately north of 55°N, corresponding to stratospheric conditions. This distribution is undoubtedly connected with the slope of the tropopause from Equator to the Pole.
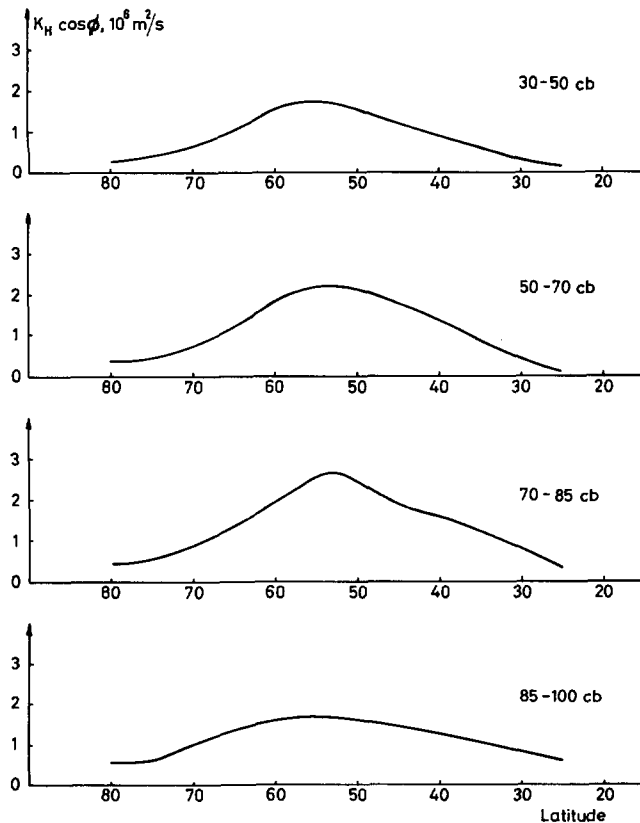
FIGURE 11.—Exchange coefficient for the transport of sensible heat as a function of latitude for the layers 30–50, 50–70, 70–85, and 85–100 cb, computed from mean annual data; units, 10^6 m/s^2.
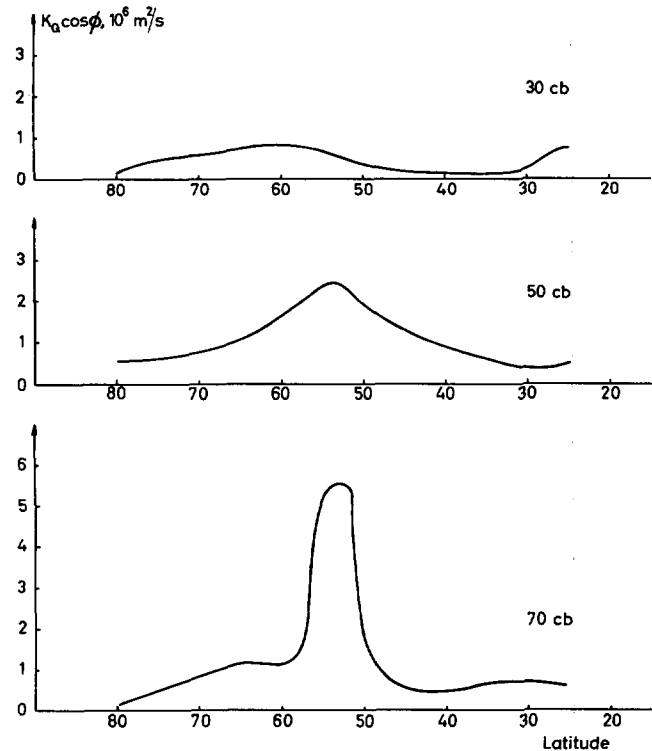

FIGURE 12.—Annual average of monthly values of the exchange coefficient for quasi-geostrophic potential vorticity as a function of latitude for the levels 30, 50, and 70 cb; units, 10^6 m^2/s.

It may further be mentioned that the absolute values of K_H tend to be rather large in middle latitudes because of the weak temperature gradients in this region. The values of K_H in the two layers 10–15 and 15–20 cb are negative at all latitudes, in agreement with the counter gradient heat transport at these elevations.

The values of K_Q and K_H discussed above were computed from annual mean data for the transport quantities and the zonal averages. One may naturally also compute values of the exchange coefficients for time periods much shorter than a year. This was done using the monthly averages for each of the 12 mo of the year. The 12 values of the exchange coefficients were then averaged with respect to time. The resulting curves are shown in figure 12 where selected values of $K_Q \cos \phi$ are plotted against latitude, while figure 13 contains the values of $K_H \cos \phi$ in a similar arrangement.

The main result of the calculations of K_Q and K_H is that an exchange coefficient approximation may be used in parameterizing the transports of potential vorticity and sensible heat in the troposphere above approximately 80 cb. Although the application of such an approximation thus is limited, it seems well-suited for investigations using a two-level quasi-geostrophic model, where the main emphasis is on tropospheric flow.
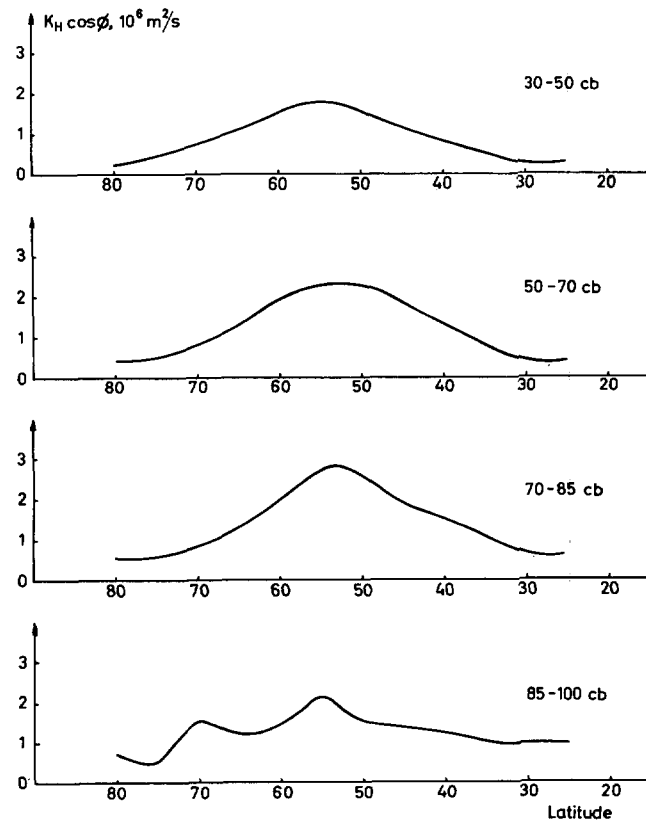

FIGURE 13.—Annual average of monthly values of the exchange coefficient for sensible heat as a function of latitude for the layers 30–50, 50–70, 70–85, and 85–100 cb; units, 10^6 m^2/s.

5. ON PARAMETERIZATION OF THE MOMENTUM TRANSPORT

The main idea behind the calculations of the transport of potential vorticity described in section 2 and the investigations of the exchange coefficients K_Q and K_H given in section 4 is to use the results for an empirical specification of the momentum transport. The goal is to specify the convergence of the momentum transport which is the dynamically important quantity in terms of the exchange coefficients and other quantities that are directly related to the zonally averaged quantities. The basic equations are eq (12), which relates the convergence of the momentum transport to the transport of potential vorticity and sensible heat, and the empirical relations (33) and (35), which define the exchange coefficients. If we at a given instant use the values of K_Q and K_H derived from atmospheric data using eq (33) and (35), we have naturally a perfect parameterization of the convergence of the momentum transport simply because we have exactly satisfied the three eq (12), (33), and (35). However, as in any other applications of an exchange coefficient, the object is to use diagnostic determinations of the exchange coefficients to find typical values of the quantities and to use them for prognostic studies of the zonally averaged quantities. One may, for example, ask if the annual means of the exchange coefficients can be used as characteristic values to study the behavior of atmospheric flow on a large time scale. We may similarly ask if the detailed specification of K_Q and K_H obtained in section 4 is necessary to obtain a parameterization of the convergence of momentum transport or, in other words, if rather simple distributions of K_Q and K_H as functions of latitude and pressure are sufficient to describe the main features of the convergence of the momentum transport. We shall try to answer the last question in this section.

Assuming that we have obtained values of K_Q and K_H, we shall investigate the specification which thereby is given for the convergence of the momentum transport. Introducing eq (33) and (34) in eq (12), we find that

$$-\frac{1}{a\cos^2\phi}\frac{\partial(uv)_z\cos^2\phi}{\partial\phi}=-K_Q\frac{\partial Q_z}{a\partial\phi}-\frac{\partial}{\partial p}\left[\frac{f_0 R}{\bar{\sigma}p}K_H\frac{\partial T_z}{a\partial\phi}\right].$$

$$(36)$$

Using further eq (34) and the thermal wind equation, we may write eq (36) in the form

$$-\frac{1}{a\cos^2\phi}\frac{\partial(uv)_z\cos^2\phi}{\partial\phi}$$
$$=-\frac{2\Omega}{a}K_Q\cos\phi+\frac{K_Q}{a^2}\frac{\partial}{\partial\phi}\left[\frac{1}{\cos\phi}\frac{\partial u_z\cos\phi}{\partial\phi}\right]$$
$$+K_Q\frac{\partial}{\partial p}\left[\frac{f_0^2}{\bar{\sigma}}\frac{\partial u_z}{\partial p}\right]-\frac{\partial}{\partial p}\left[\frac{f_0^2}{\bar{\sigma}}K_H\frac{\partial u_z}{\partial p}\right]. \quad (37)$$

We note from eq (37) that, when the equation is multiplied by $\cos^2\phi$ and integrated from $-\pi/2$ (the South Pole)

to $\pi/2$ (the North Pole), we obtain zero from the left side of the equation. In general, we have not specified K_Q and K_H in such a way that the corresponding integral of the right-hand side of eq (37) will vanish. The parameterization of the convergence of the momentum should therefore be corrected to satisfy this condition. If the right-hand side of eq (37) is denoted by $S(\phi, p, t)$, we may write the corrected parameterization in the form

$$-\frac{1}{a\cos^2\phi}\frac{\partial(uv)_z\cos^2\phi}{\partial\phi}=S-\frac{\tilde{S}}{a\cos^2\phi} \quad (38)$$

where

$$\tilde{S}=\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}Sa\cos^2\phi\,d\phi. \quad (39)$$

To illustrate the behavior of the parameterization of the momentum transport, we shall select a particularly simple example. We select a quasi-geostrophic two-level model in Cartesian geometry, which means that we will use a beta-plane approximation. The region of interest will be a channel bounded by solid walls to the north and to the south. The potential vorticity at the upper level (25 cb) in this model is

$$Q_1=f+\xi_1-q^2\psi_T \quad (40)$$

where the subscript 1 denotes quantities at the level 25 cb and where

$$q^2=\frac{2f_0^2}{\sigma P^2} \quad (41)$$

is the stability parameter. In this expression, f_0 is a standard value of the Coriolis parameter; $P=50$ cb; $\sigma=-(\alpha/\theta)$ $(\partial\theta/\partial p)$ is a measure of static stability; α, the specific volume; and θ, the potential temperature. Furthermore, ψ_T is the thermal stream function defined as half the difference between the stream functions at the upper (25-cb) and lower (75-cb) levels.

The potential vorticity at the lower level is

$$Q_3=f+\xi_3+q^2\psi_T. \quad (42)$$

We now obtain the following two equations corresponding to eq (38):

$$-\frac{\partial M_1}{\partial y}=S_1-\tilde{S}_1$$

and

$$-\frac{\partial M_3}{\partial y}=S_3-\tilde{S}_3 \quad (43)$$

where

$$M_1=(u_1v_1)_z,$$
$$M_3=(u_3v_3)_z,$$

$$S_1=-\beta K_1+K_1\frac{\partial^2 u_{1z}}{\partial y^2}+q^2(K_H-K_1)u_{Tz}, \quad (44)$$

and

$$S_3=-\beta K_3+K_3\frac{\partial^2 u_{3z}}{\partial y^2}+q^2(K_3-K_H)u_{Tz}. \quad (45)$$

In eq (44) and (45), K_1 and K_3 are the values of K_Q at levels 1 and 3, respectively, while K_H is the value of

the exchange coefficient for heat at level 2 (50 cb). In the beta-plane approximation, we have

$$\tilde{S} = \frac{1}{D}\int_0^D S \, dy \tag{46}$$

where D is the width of the channel.

To show that the gross characteristics of the momentum transport can be reproduced without knowing all the details of the distributions of K_Q and K_H with latitude, we shall assume for purposes of demonstration that K_1, K_3, and K_H are constants. If we further restrict the wind profiles to those cases where $\partial u_z/\partial y = 0$ at both walls, we find that

$$\tilde{S}_1 = -\beta K_1 + q^2(K_H - K_1)\tilde{u}_{Tz} \tag{47}$$

and

$$\tilde{S}_3 = -\beta K_3 + q^2(K_3 - K_H)\tilde{u}_{Tz}; \tag{48}$$

therefore,

$$-\frac{\partial M_1}{\partial y} = K_1\frac{\partial^2 u_{1z}}{\partial y^2} + q^2(K_H - K_1)(u_{Tz} - \tilde{u}_{Tz}) \tag{49}$$

and

$$-\frac{\partial M_3}{\partial y} = K_3\frac{\partial^2 u_{3z}}{\partial y^2} + q^2(K_3 - K_H)(u_{Tz} - \tilde{u}_{Tz}). \tag{50}$$

Observational studies of momentum transport show that there is, on the average, a convergence of the momentum transport in middle latitudes and a divergence in the low and high latitude and that these patterns are the same at all levels in the troposphere, but with larger values of the convergence and divergence in the upper troposphere (Wiin-Nielsen et al. 1963). We note from eq (39) and (40) that the first term in the expression in general will give a contribution that is opposite to these patterns because the second derivative will be negative in middle latitudes and positive in the low and high latitudes for a typical jetlike profile of the zonal wind. The major contribution must therefore come from the second term in expressions (39) and (40). Since $(u_{Tz} - \tilde{u}_{Tz})$ in general will be positive in middle latitudes and negative outside this region, it is a necessity that $K_1 < K_H$ and $K_3 > K_H$. When using the data obtained and displayed in section 4 (figs. 10 and 11), K_1 was computed as an average value of K_Q at 30 cb; K_3 was obtained in a similar way from the data at 70 cb, while K_H was obtained as a latitudinal average of the mean values of K_H for the layers 30–50 and 50–70 cb. The values thus obtained were: $K_1 = 0.87 \times 10^6$ m²/s, $K_3 = 1.97 \times 10^6$ m²/s, and $K_H = 1.73 \times 10^6$ m²/s. It is thus seen that these empirically derived values satisfy the necessary requirements given above.

We note furthermore from eq (49) and (50) that the quantity $(u_{Tz} - \tilde{u}_{Tz})$ for a typical profile of u_{Tz} will be positive in middle latitudes and negative elsewhere in the hemisphere. The last terms in the two expressions will therefore act to give convergence of the momentum transport in middle latitudes and divergence elsewhere, but will in general be counteracted by the first terms in eq (39) and

(40). For sufficiently large values of u_{Tz} (i.e., when the baroclinicity of the atmosphere is strong), we obtain the typical distribution of the convergence of the momentum transport, while small values of u_{Tz} (i.e., when we approach a barotropic state of the mean flow), there is a possibility that the distribution of the convergence of the momentum transport may be reversed, as it indeed would be if the atmosphere were operating in a barotropic mode.

To make some simple estimates, we may assume that the distributions of the zonal winds are given by the expressions

$$u_{1z} = \tfrac{1}{2}U_1(1 - \cos 2\lambda y), \tag{51}$$

$$u_{3z} = \tfrac{1}{2}U_3(1 - \cos 2\lambda y), \qquad \lambda = \frac{\pi}{D}, \tag{52}$$

and

$$u_{Tz} = \tfrac{1}{2}U_T(1 - \cos 2\lambda y) \tag{53}$$

where

$$U_T = \tfrac{1}{2}(U_1 - U_3). \tag{54}$$

These distributions satisfy the requirements that $(\partial u/\partial y) = 0$ for $y = 0$ and $y = D$ as assumed above. We find that

$$\tilde{u}_{Tz} = \tfrac{1}{2}U_T, \tag{55}$$

and the expressions (49) and (50) become

$$-\frac{\partial M_1}{\partial y} = -A_1\cos 2\lambda y \tag{56}$$

and

$$-\frac{\partial M_3}{\partial y} = -A_3\cos 2\lambda y \tag{57}$$

where

$$A_1 = \tfrac{1}{2}q^2U_T(K_H - K_1) - 2\lambda^2 K_1 U_1 \tag{58}$$

and

$$A_3 = \tfrac{1}{2}q^2U_T(K_3 - K_H) - 2\lambda^2 K_3 U_3 \tag{59}$$

or

$$A_1 = \{\tfrac{1}{2}q^2(K_H - K_1) - 2\lambda^2 K_1\}U_T - 2\lambda^2 K_1 U_* \tag{60}$$

and

$$A_3 = \{\tfrac{1}{2}q^2(K_3 - K_H) + 2\lambda^2 K_3\}U_T - 2\lambda^2 K_3 U_* \tag{61}$$

where

$$U_* = \tfrac{1}{2}(U_1 + U_3). \tag{62}$$

It is seen from eq (56) that, to have the normal distribution of $-(\partial M_1/\partial y)$, we must have $A_1 > 0$. This happens according to eq (60) when $U_T > U_{TC1}$, where

$$U_{TC1} = \frac{2\lambda^2 K_1}{\frac{1}{2}q^2(K_H - K_1) - 2\lambda^2 K_1} U_*, \qquad (63)$$

while the distribution of $-(\partial M_3/\partial y)$ is normal when $A_3 > 0$ or when $U_T > U_{TC3}$ where

$$U_{TC3} = \frac{2\lambda^2 K_3}{\frac{1}{2}q^2(K_3 - K_H) + 2\lambda^2 K_3} U_*. \qquad (64)$$

When we use the values of K_1, K_3, and K_H given above and $q = 2 \times 10^{-6}$ m^{-1} and $D = 10^7$ m, we find $U_{TC1} = 0.11 U_*$ and $U_{TC3} = 0.45 U_*$. Since the conditions $U_T > U_{TC1}$ and $U_T > U_{TC3}$ normally are satisfied in the atmosphere since $U_T \simeq 0.5 U_*$, we find that the parameterization given here for $-\partial M/\partial y$ will give a distribution which agrees with the observed distribution, but that the possibility exists that the convergence of the momentum transport may be reversed if it happens that U_T is sufficiently small.

The data from the year 1963 can be used to obtain characteristic values of U_1, U_3, and U_T. Recalling that these values represent the maximum zonal wind at the various levels, we find from the annual mean winds that $U_1 \simeq 26$ m/s and $U_3 \simeq 8$ m/s. Consequently, $U_T \simeq 9$ m/s. Using these values of the winds and the values quoted above for the other parameters, we find that $A_1 = 11.06 \times 10^{-6}$ m/s^2 and $A_3 = 1.20 \times 10^{-6}$ m/s^2, which shows that the convergence of the momentum transport is much larger at the upper level than at the lower level, in agreement with observations.

The facts that K_Q becomes negative in the atmospheric boundary layer and that K_H is negative in the lower stratosphere seem to limit the approach described here to a two-level model. Further investigations will be necessary to extend the use of this form of lateral mixing to a higher vertical resolution.

6. SUMMARY AND CONCLUSIONS

The main purpose of this paper has been to report the results of diagnostic studies of the meridional transport of quasi-geostrophic potential vorticity. The transport was computed at six different levels in the vertical direction from known values of the transports of relative momentum and sensible heat (1963 data). The transport of potential vorticity is negative (southward) in the major part of the troposphere above about 80 cb, while it is positive (northward) in the lowest layer near the ground. A small region of positive (northward) transport is found near the subtropical jet stream at 20 cb and 40°N in the summer season.

The computed values of the transport of quasi-geostrophic potential vorticity were used to make a diagnostic calculation of the heat sources in the atmosphere using a quasi-geostrophic model of the dynamics of the atmosphere. The results of this calculation show that the atmosphere is heated south of 50°N and cooled north of this latitude. The maximum heating and cooling takes place in the lower part of the troposphere around 70 cb. The derived heating agrees favorably with other estimates of the atmospheric heat sources based on heat budget calculations, although the intensity of the heating south of 50°N is somewhat larger than the heating determined from the budget considerations. It is pointed out that the difference may be due to the less intense mean meridional circulation that occurs in a quasi-geostrophic model as compared with the real atmosphere. The mean meridional circulation for the annual mean was also computed and shows good agreement with the mean meridional circulation derived by other investigators.

The computed values of the transport of potential vorticity were further used to investigate the relation between the transport and the meridional gradient of the potential vorticity itself. Outside the atmospheric boundary layer, it was found that the transport in general is along the gradient of potential vorticity and that it therefore is possible to describe the process, using a coefficient of lateral mixing. These coefficients were calculated empirically from the data as a function of latitude and pressure. The distributions show in general a maximum in middle latitudes, while the vertical variation is such that the exchange coefficient decreases with decreasing pressure.

An exchange coefficient for the transport of sensible heat was also computed from the data as a function of latitude and pressure because numerical values of this quantity are necessary for later studies. The results show that the exchange coefficient is positive in the troposphere and negative in the stratosphere. The distributions, furthermore, show a maximum in the region 50°–60°N, while the values decrease rather slowly with decreasing pressure in the troposphere.

The derived values of the exchange coefficients were finally used to formulate a parameterization of the convergence of the momentum transport, and it was shown that the formula reproduces the gross features of the observed distribution. The use of the proposed parameterization is probably restricted to atmospheric models with a low vertical resolution due to the fact that the heat transport is against the gradient of the temperature in the lower stratosphere, while the transport of potential vorticity is against the gradient of the potential vorticity in the lowest troposphere. Even with these limitations, however, it may be useful to apply the proposed parameterizations to studies of the atmospheric circulation, especially studies concerned with the behavior of the atmospheric flow on a rather large time scale. The results of such a study will be reported in a separate paper.

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